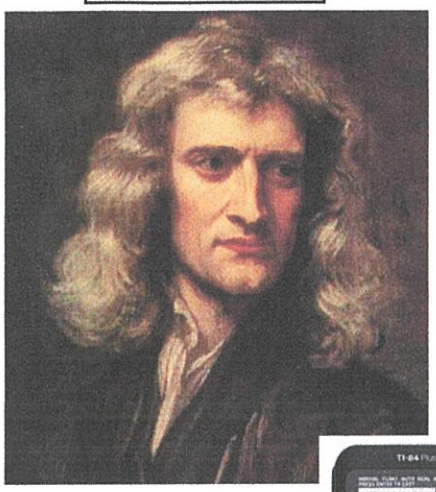
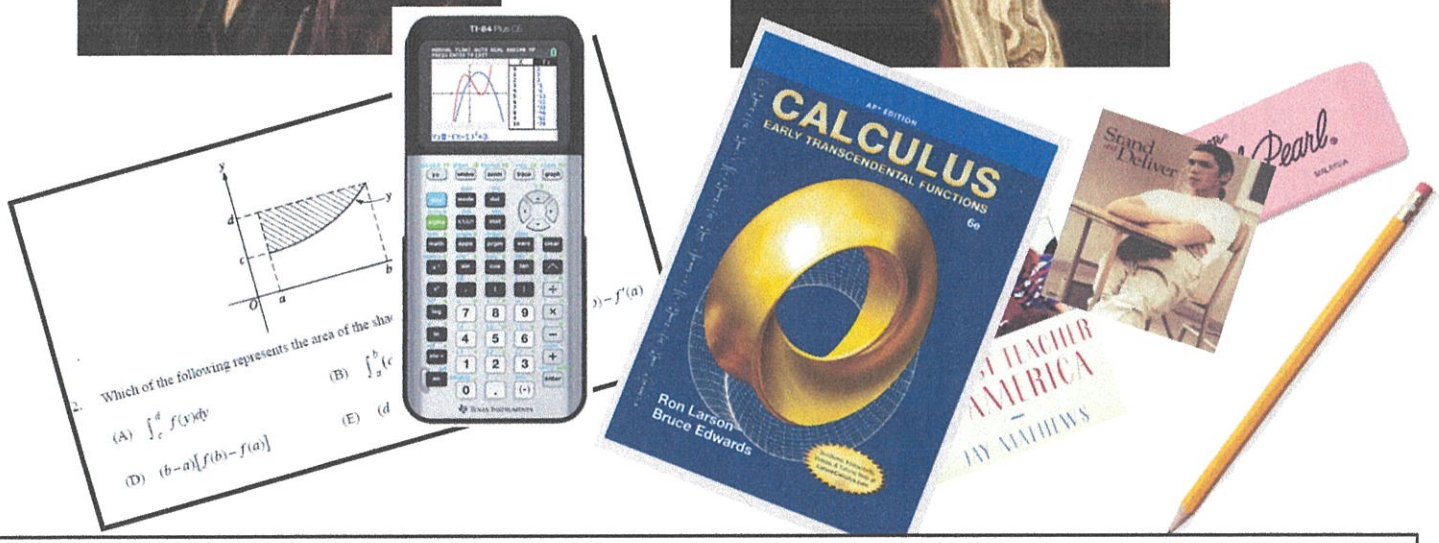
Oak Park High School
AP Calculus BC

Isaac Newton



Gottfried Wilhelm Leibniz



Summer Packet – 2024

Be sure to show all your work. Credit will not be given for answers not supported by adequate work.

Name _____ Period _____

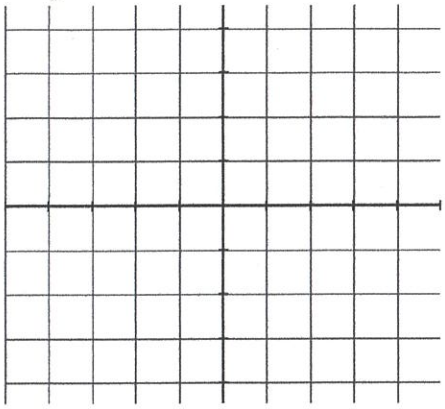
Due Monday August 12, 2024

Post any questions to the Google Classroom **Code: vita5v2**

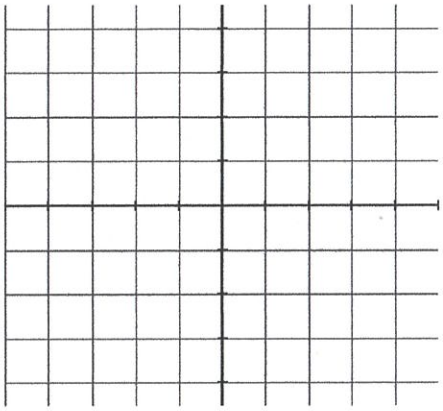
Topic C: Graphs of Common Functions

Sketch each of the following as accurately as possible. **You will need to be VERY familiar with each of these graphs throughout the year**. You may use a graphing calculator for some of them if you have access to one over the summer. If you do not have one for the summer, I strongly recommend you use try www.desmos.com. There is an app for Desmos as well that is free that you can install on your phones. Again, these are VERY important graphs to know. Be very accurate with regards to "open circles" and "closed circles" as those features may not be revealed on a graphing utility.

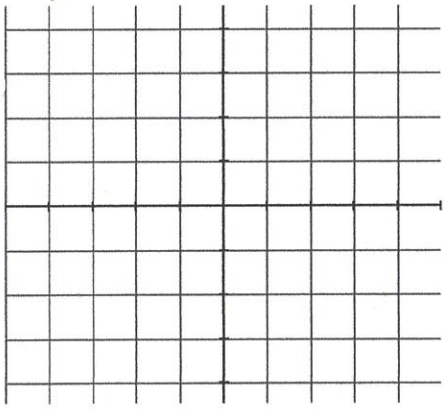
1. $y = x$



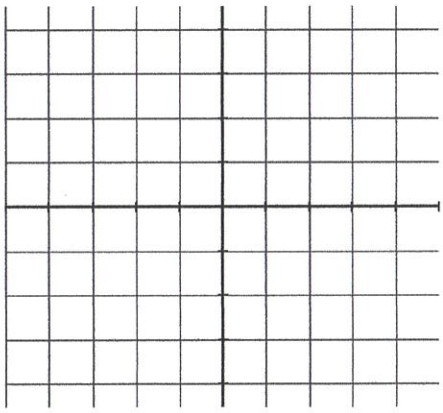
2. $y = x^2$



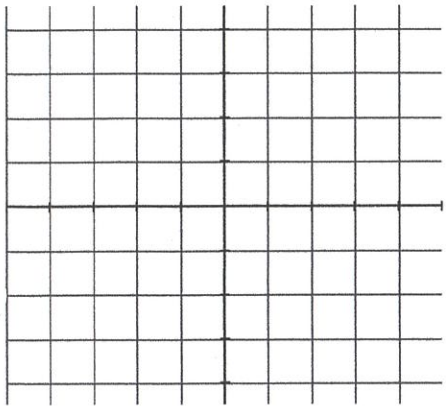
3. $y = x^3$



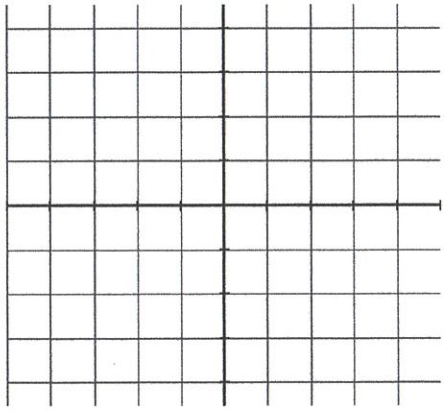
4. $y = \sqrt{x}$



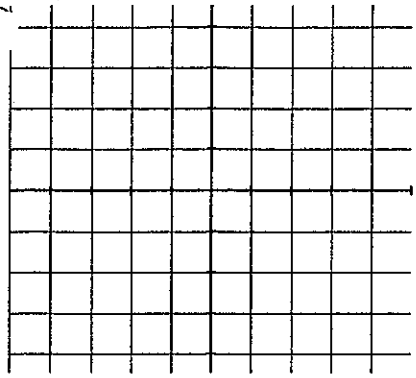
5. $y = |x|$



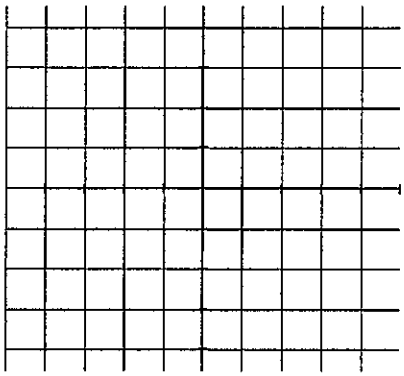
6. $y = \dfrac{|x|}{x}$



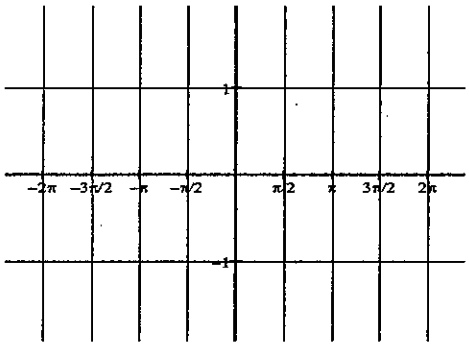
7. $y = x^{1/3}$



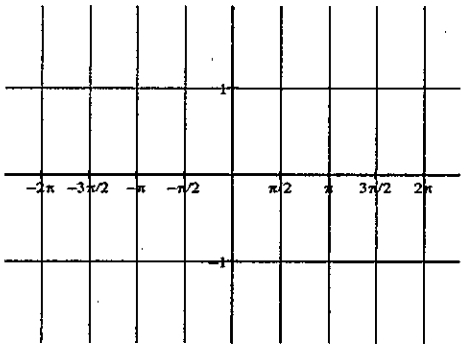
8. $y = x^{2/3}$



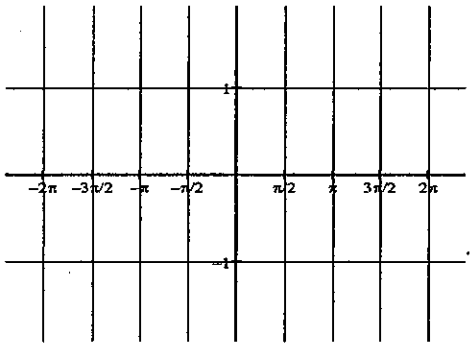
9. $y = \sin x$



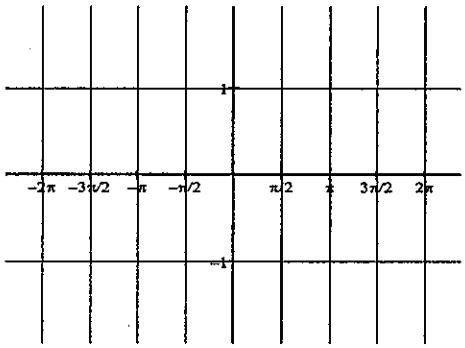
10. $y = \cos x$



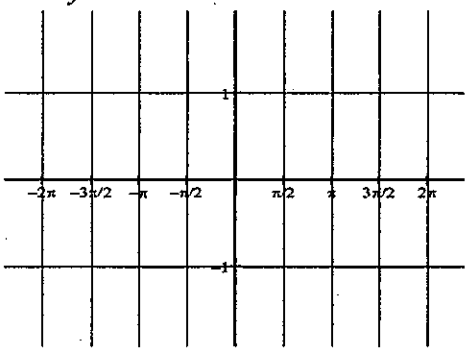
11. $y = \tan x$



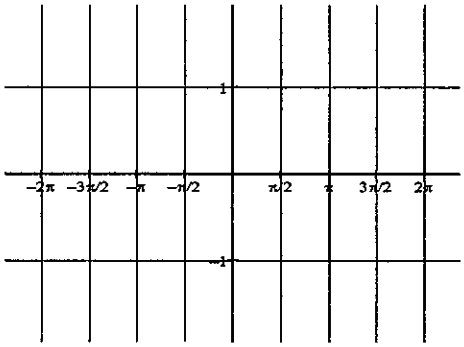
12. $y = \cot x$



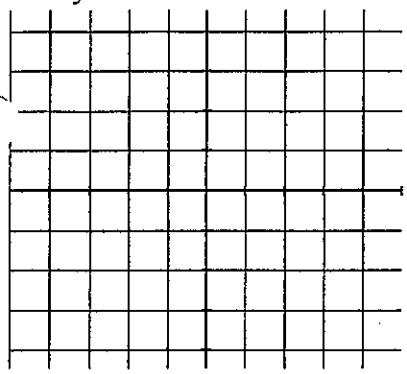
13. $y = \sec x$



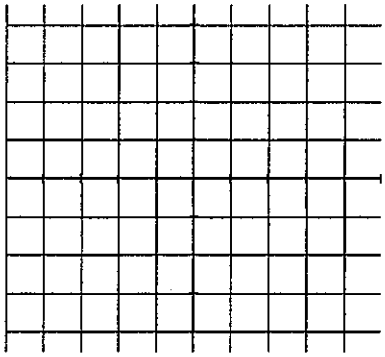
14. $y = \csc x$



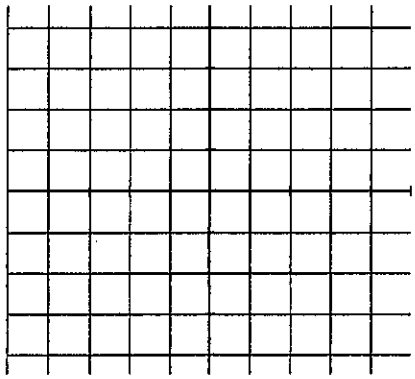
15. $y = e^x$



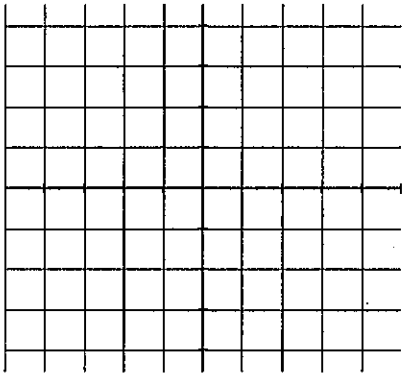
16. $y = \ln x$



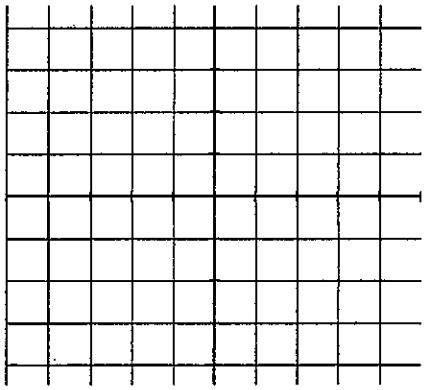
17. $y = \dfrac{1}{x}$



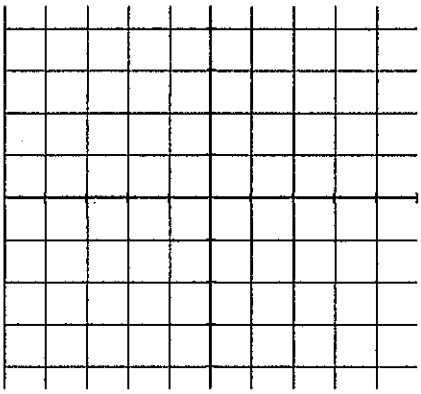
18. $y = [\![x]\!]$



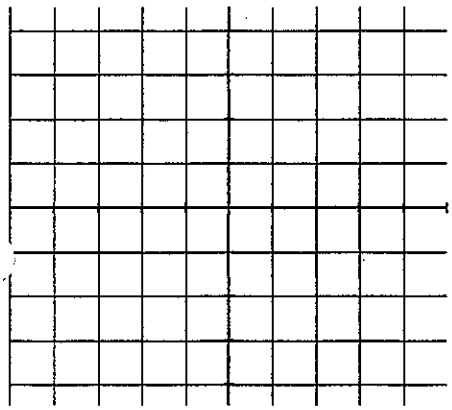
19. $y = \dfrac{1}{x^2}$



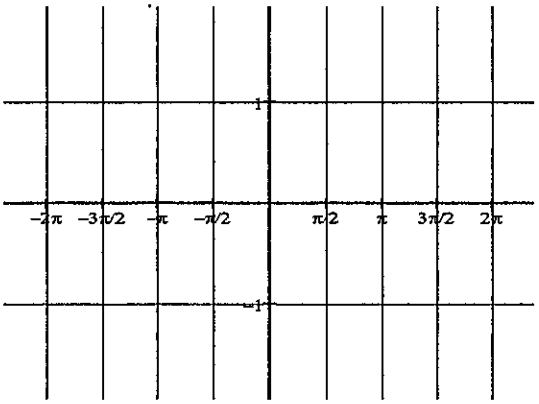
20. $y = 2^x$



21. $y = \sqrt{4 - x^2}$



22. $y = \dfrac{\sin x}{x}$



Topic E: Function Transformations

If $f(x)=x^2-1$, describe in words what the following would do to the graph of $f(x)$:

1.) $f(x)-4$

2.) $f(x-4)$

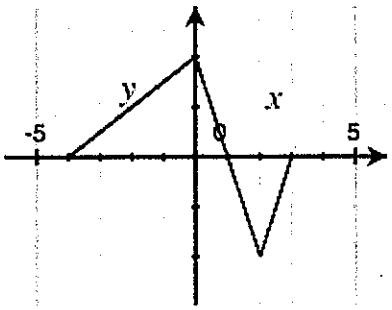
3.) $-f(x+2)$

4.) $5f(x)+3$

5.) $f(2x)$

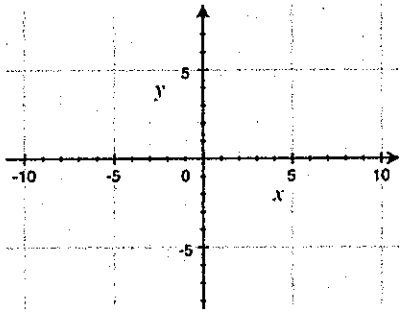
6.) $|f(x)|$

Here is a graph of $y=f(x)$:

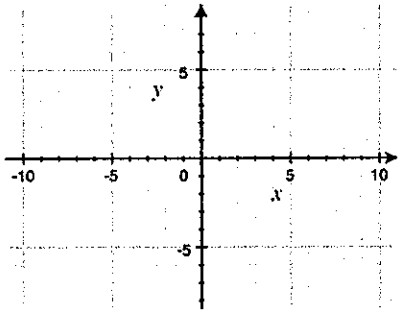


Sketch the following graphs:

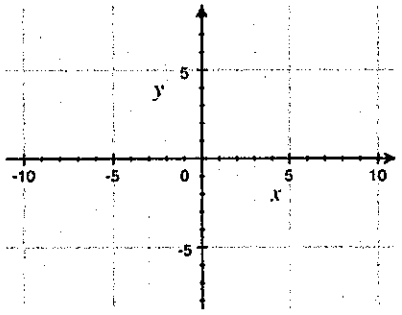
7.) $y=2f(x)$



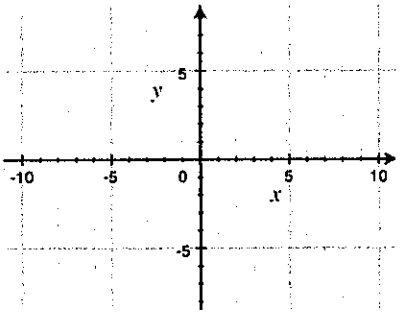
8.) $y=-f(x)$



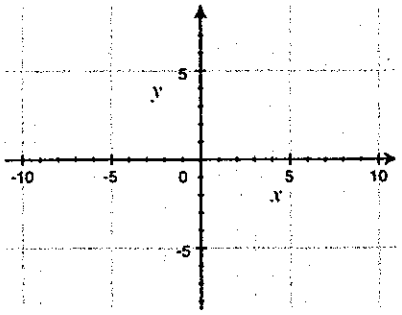
9.) $y=f(x-1)$



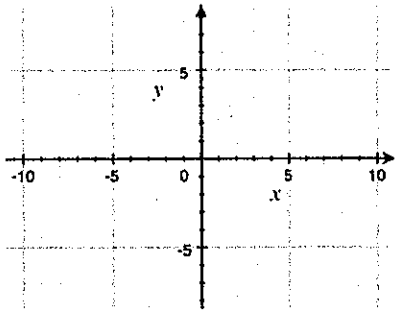
10.) $y=f(x)+2$



11.) $y=|f(x)|$



12.) $y=f(|x|)$



Topic I: Asymptotes

For each function, find the equations of both the vertical asymptote(s) and horizontal asymptote (if it exists) and the location of any holes.

1.) $y=\frac{x-1}{x+5}$

2.) $y=\frac{8}{x^2}$

3.) $y=\frac{2x+16}{x+8}$

4.) $y=\frac{2x^2+6x}{x^2+5x+6}$

5.) $y=\frac{x}{x^2-25}$

6.) $y=\frac{x^2-5}{2x^2-12}$

7.) $y=\frac{x^3}{x^2+4}$

8.) $y=\frac{x^3+4x}{x^3-2x^2+4x-8}$

9.) $y=\frac{10x+20}{x^3-2x^2-4x+8}$

10.) $y=\frac{1}{x}-\frac{x}{x+2}$ (Hint: Express with a common denominator)

Topic P: Exponential Functions and Logarithms

Simplify the following:

1.) $\log_2 \frac{1}{4}$

2.) $\log_8 4$

3.) $\ln \frac{1}{\sqrt[3]{e^2}}$

4.) $5^{\log_5 40}$

5.) $e^{\ln 12}$

6.) $\log_{12} 2 + \log_{12} 9 + \log_{12} 8$

7.) $\log_2 \frac{2}{3} + \log_2 \frac{3}{32}$

8.) $\log_{\frac{1}{3}} \frac{4}{3} - \log_{\frac{1}{3}} 12$

9.) $\log_3 \left(\sqrt{3}\right)^5$

Solve the following:

10.) $\log_5 (3x-8) = 2$

11.) $\log_9 (x^2 - x + 3) = \frac{1}{2}$

12.) $\log(x-3) + \log 5 = 2$

13.) $\log_2 (x-1) + \log_2 (x+3) = 5$

14.) $\log_5 (x+3) - \log_5 x = 2$

15.) $\ln x^3 - \ln x^2 = \frac{1}{2}$

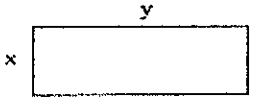
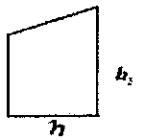
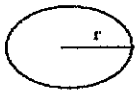
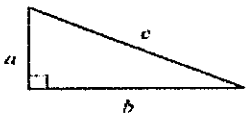
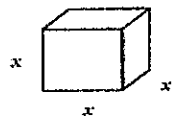
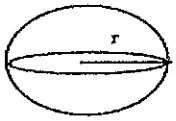
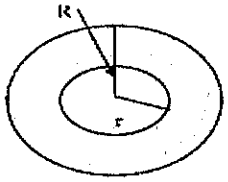
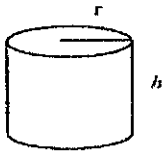
16.) $3^{x-2} = 18$

17.) $e^{3x+1} = 10$

18.) $8^x = 5^{2x-1}$

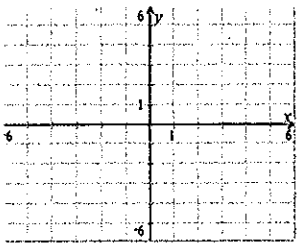
Topic Q: Geometry

(.) You will use each of the following formulas in AP Calculus BC. Complete each of the following.

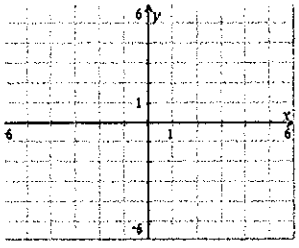
Square  Perimeter = _____ Area = _____	**Rectangle**  Perimeter = _____ Area = _____	**Trapezoid**  Area = _____
Circle  Circumference = _____ Area = _____	**Triangle**  Pythagorean Theorem (only good for right triangles) = _____ Area (of any triangle) = _____	**Cube**  Volume = _____ Surface Area = _____
Sphere  Volume = _____	**"Washer"**  Area of the shaded region = _____	**Cylinder**  Volume = _____

Find the area between the x-axis and $f(x)$ from $x = 0$ to $x = 5$. Sketch the region to verify.

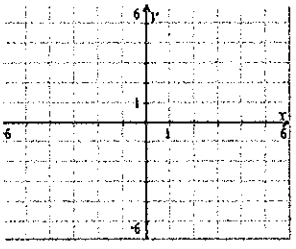
2.) $f(x) = 4$



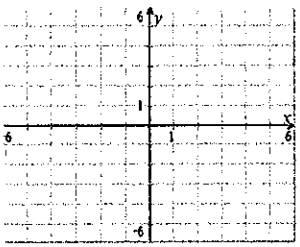
3.) $f(x) = x$



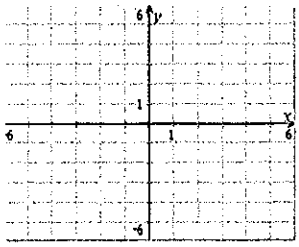
4.) $f(x) = x + 3$



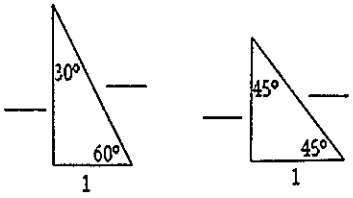
5.) $f(x) = \sqrt{9 - x^2}$



6.) $f(x) = \begin{cases} x+1, & x \le 2 \\ 5-x, & x > 2 \end{cases}$



7.) Fill in the four blanks.



Topic R: Basic Right Angle Trigonometry

Solve the following:

If point P is on the terminal side of θ, find all 6 trigonometric functions of θ. (Answers need not be rationalized.)

1.) $P(-2,4)$

2.) $P(\sqrt{5},-2)$

3.) If $\cos\theta = -\frac{5}{13}$, in quadrant II, find $\sin\theta$ and $\tan\theta$.

4.) If $\cot\theta = \frac{2\sqrt{10}}{3}$, in quadrant III, find $\sin\theta$ and $\cos\theta$.

5.) State the quadrant in which each of the following is true.

a.) $\sin\theta > 0$ and $\cos\theta < 0$

b.) $\csc\theta < 0$ and $\cot\theta > 0$

c.) $\tan\theta > 0$ and $\sec\theta < 0$

Topic U: Solving Trigonometric Equations

Solve each equation on the interval $[0,2\pi)$. Do not use a calculator.

1.) $\sin^2 x = \sin x$

2.) $3\tan^3 x = \tan x$

3.) $\sin^2 x = 3\cos^2 x$

4.) $\cos x + \sin x \tan x = 2$

5.) $\sin x = \cos x$

6.) $2\cos^2 x + \sin x - 1 = 0$

Topic V. Graphical Solutions to Equations and Inequalities

You have a shiny new graphing calculator. So when are we going to use it? A graphing calculator is a tool that is required on the AP Calculus exam. About 25% of the exam, a calculator is permitted. It is vital you are comfortable using it. According to College Board the following skills will be required on the AP exam: plot the graph of a function, find the zeros of functions (solve equations numerically), numerically calculate the derivative of a function, and numerically calculate the value of a definite integral. I have created a couple videos showing the basics including graphing, finding zeros (x-intercepts), finding minimums/maximums, intersections of graphs, and derivatives. I will be conducting lessons on a TI-84 Plus CE. If you plan to use another type of calculator, you may need to explore YouTube or other resources for details on using your calculator.

Use your graphing calculator to find the zeros of each of the following functions. Make sure each equation is set equal to zero first.

1.) $3x^3 - x - 5 = 0$

2.) $2x^2 - 1 = 2^x$

3.) $2\ln(x+1) = 5\cos x \text{ on } [0, 2\pi)$

Use your graphing calculator to find the solution (intersection) of the given system of equations.

4.) $\begin{cases} f(x) = x^4 - 6.5x^2 + 6x + 2 \\ g(x) = 1 + x + e^{x^2 - 2x} \end{cases}$

Use your graphing calculator to find both a relative maximum and a relative minimum point of the given function.

5.) $h(x) = 2x^5 - 3x^4 + x - 4$

Oak Park High School **Name** _____

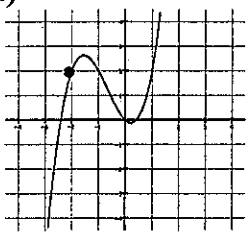
AP Calculus BC

Unit 2 Review

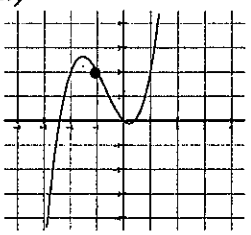
DIFFERENTIATION – Definition of Derivative and Basic Derivative Rules

Determine whether the slope of the tangent line to the curve at the indicated point is positive, negative or zero.

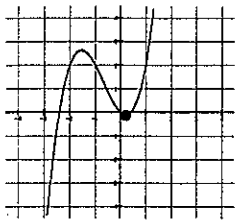
1.)



2.)



3.)



3.) If $f(2)=3$ and $f'(2)=-1$, find the equation of the tangent line when $x=2$.

4.) Find the equation of the tangent line to the graph of $f(x)=x^2-2x-3$ when $x=2$.

5.) Differentiate $y=\dfrac{3x}{x^2+1}$

6.) For the function $f(t)=\dfrac{t^3+2}{t}$, find the following.

a.) the average rate of change of $f(t)$ on the interval $[1, 4]$

b.) the instantaneous rate of change of $f(t)$ when $t=2$

7.) Suppose that $h(x)=\dfrac{g(x)}{f(x)}$ and $g(2)=3,\ g'(2)=-1,\ f(2)=5,\ f'(2)=-2$.
Find $h'(2)$.

8.) Let $f(x)=\begin{cases} ax, & x\le 1 \\ bx^2+x+1, & x>1 \end{cases}$. Find all possible values of a and b such that $f(x)$ is differentiable at $x=1$.
Show proper justification.

Skip

9.) If $f(x)=2x^2+4$, which of the following will calculate the derivative of $f(x)$?

(A) $\dfrac{[2(x+\Delta x)^2+4]-(2x^2+4)}{\Delta x}$

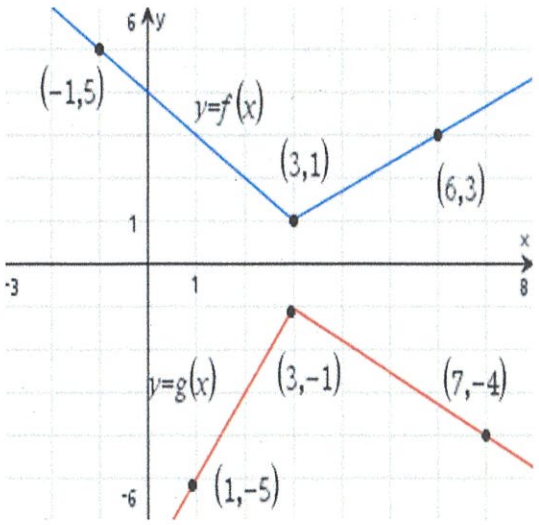
(B) $\lim\limits_{\Delta x\to 0}\dfrac{(2x^2+4+\Delta x)-(2x^2+4)}{\Delta x}$

(C) $\lim\limits_{\Delta x\to 0}\dfrac{[2(x+\Delta x)^2+4]-(2x^2+4)}{\Delta x}$

(D) $\dfrac{(2x^2+4+\Delta x)-(2x^2+4)}{\Delta x}$

10.) Let $h(x)=f(x)\cdot g(x)$.
Find $h'(2)$.

(A) -2 **(B)** 1
(C) 7 **(D)** -1



11.) What is $g'(3)$?

Oak Park High School **Name** _____

AP Calculus BC

Unit 3 Review

DIFFERENTIATION – Compositions, Implicit and Inverse Functions

1.) Find $\frac{dy}{dx}$ for $y = x^3\sqrt{x+1}$ and simplify.	**2.)** Find $f'(x)$ if $f(x) = \cos^5(4x)$
3.) Find $f'(\theta)$ if $f(\theta) = \sqrt{\sin(2\theta)}$	**4.)** Given $f(x) = x^2(2x-5)^2$, find $f'(x)$ and simplify.

5.) Find $\frac{d^2y}{dx^2}$ for $x^2+xy+y^2=3$

6.) Find y'' for $y=\frac{\csc x}{2}$

7.) Find y' for $y=\ln\sqrt{x^2-4x-7}$

8.) Find $h'(x)$ for $h(x)=\ln\frac{x(x-1)^3}{\sqrt{x-2}}$. You do not have to simplify.

9.) Find y' for $y=-4e^{\sec x}$

10.) Find y' for $y=3^{5x}$

22.) Find the derivative of the function $f(x)=\arccos\left(3x^2-1\right)$.

(A) $f'(x)=-\dfrac{6x}{\sqrt{1+\left(3x^2-1\right)^2}}$ **(B)** $f'(x)=-\dfrac{3x^2-1}{\sqrt{1-\left(3x^2-1\right)^2}}$

(C) $f'(x)=-\dfrac{6x}{\sqrt{\left(3x^2-1\right)^2-1}}$ **(D)** $f'(x)=-\dfrac{6x}{\sqrt{1-\left(3x^2-1\right)^2}}$

23.) Find the derivative of the function $f(x)=x^2\cdot\arctan(5x)$.

(A) $f'(x)=\dfrac{10x}{1+25x^2}$ **(B)** $f'(x)=\dfrac{10x^2}{1+25x^2}$

(C) $f'(x)=2x\arctan(5x)+\dfrac{5x^2}{1+25x^2}$ **(D)** $f'(x)=2x\arctan(5x)+\dfrac{5x^2}{1+5x^2}$

24.) Let $f(x)=\arcsin x-2x$.

a.) Find $f'(x)$.

b.) Find the equation of the tangent line to $f(x)$ at the point where $x=0$.